Calculus Warm Up # 7-5

FR practice:

Pick up a sheet on the stool.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

Due Today

Classwork

WU's (2 weeks)

2009 FR

BC - FR Practice (purple)

HW Quiz by group

10.8, p. 623 (2 days)

10.9, p. 630

12.5, p. 715

Pink FR practice
(with calculator)

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

   (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

   (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

   (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

   (d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

(no calculator)

6. The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the nth derivative of f at $x = 1$ is given by $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \ge 2$.

   (a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

   (b) The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.

   (c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.

   (d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.

Today's Classwork

Pink BC - FR practice (back #6 is no calculator.)

HW: Finish Pink FR and find a quiet time and place to time yourself on the MC practice.

Check answers (following this slide)

for Tan FR, Pink FR, and last MC practice.

Monday: Bring your questions!

Classwork: FR practice (tan) answers.

*No Calculator

2016 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

4. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y. $\frac{d^2y}{dx^2} = 2x - \frac{x^2}{2} + \frac{y}{4}$

(b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer. Show $f' = 0$ & $f'' = -$; concave down confirms max

(c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \to -1}\left(\frac{g(x) - 2}{3(x+1)^2}\right)$. Show the work that leads to your answer. $\boxed{-\frac{1}{3}}$

(d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.

$h(1) = \frac{5}{4}$

backside of the tan FR practice

2016 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

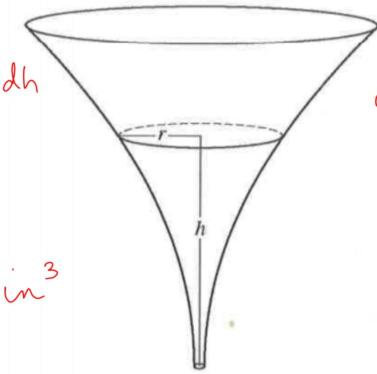
★No Calculator

a) $\text{Avg Value} = \frac{1}{10}\int_0^{10} r(h)\,dh$

$= \frac{109}{60}$ in.

b) $V = \frac{2209}{40}\pi$ in^3

c) $-\frac{2}{3}$ in/sec



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

Pink FR practice answers
(with calculator)

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(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

a) $R'(2) \approx -120$ liters/hr^2

b) $\int_0^8 R(t)\,dt \approx 8050$ liters

c) $\approx 49{,}786$ liters

d) when $t = 3$, $W(3) - R(3) \approx 325$ liters/hr.

when $t = 6$, $W(6) - R(6) \approx -409$ liters/hr.

Intermediate Value Th. guarentees at least one time t where $W(t) - R(t) = 0$

(no calculator)

6. The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the nth derivative of f at $x = 1$ is given by $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \geq 2$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

(b) The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.

(c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.

(d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.

a) $1 - \frac{1}{2}(x-1) + \frac{1}{4}\frac{(x-1)^2}{2!} + \frac{1}{4}\frac{(x-1)^3}{3!}$

n^{th} term: $\frac{(-1)^n(x-1)^n}{2^n \cdot n}$

b) interval: $-1 < x < 3$

endpts: $x = -1$ divergent p-series

$x = 3$ Alternating harmonic series converges

Interval of Conv: $-1 < x \leq 3$

c) $f(1.2) \approx \frac{181}{200}$

d) error $< \left| -\frac{1}{4}\frac{(0.2)^3}{3!} \right| < \frac{1}{24} \cdot \frac{1}{125}$

error $< \frac{1}{3000}$

$\frac{1}{3000} < 0.001$

Last BC - MC practice

1. A	9. A	16. D	24. C
2. C	10. A	17. B	25. A
4. D	11. B	18. D	26. D
5. C	12. D	19. B	27. C
6. A	13. D	20. C	28. B
7. A	14. D	21. D	29. B
8. B	15. C	23. C	30. C

BC - FR Practice (purple) Name
Classwork

No calculator is allowed for problems on this part of the exam.

1. Consider the function f given by $f(x) = xe^{-2x}$ for all $x \geq 0$.

(A) Find $\lim_{x \to \infty} f(x)$.

(B) Find the maximum value of f for $x \geq 0$. Justify your answer.
→ outcome

(C) Evaluate $\int_0^\infty f(x)\,dx$, or show that the integral diverges.

A) $\lim_{x \to \infty} f(x) = 0$

B) $\frac{1}{2e}$

C) $\frac{1}{4}$

Points
A (1) answer
(2) $f'(x)$
B (1) identify $x = \frac{1}{2}$
(1) justification
C (2) Antiderivative
(1) $\lim_{b \to \infty}$
(1) answer

(purple - backside)

2. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n (n+1)} = 1 + \frac{x-2}{3 \cdot 2} + \frac{(x-2)^2}{3^2 \cdot 3} + \frac{(x-2)^3}{3^3 \cdot 4} + \cdots + \frac{(x-2)^n}{3^n (n+1)} + \cdots$$

for all real numbers x for which the series converges.

(A) Determine the interval of convergence of the power series for f. Show the work that leads to your answer.

(B) Find the value of $f''(2)$.

(C) Use the first three nonzero terms of the power series for f to approximate $f(1)$. Use the alternating series error bound to show that this approximation differs from $f(1)$ by less than $\frac{1}{100}$.

A) $-1 \leq x < 5$

B) $f''(2) = \frac{2}{27}$ → (1) point correct answer

C) $f(1) \approx \frac{47}{54}$, error $< \frac{1}{108}$
$\frac{1}{108} < \frac{1}{100}$

points:
A (1) set up ratio
(1) find limit of ratio
(1) $-1 < x < 5$
(1) endpts
(1) correct answer & analysis

points:
(1) Approximation
(1) error < |4th term|
(1) analysis